Collateral and Punishment: Coexistence in General Equilibrium

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Abstract

This paper builds a general equilibrium model where both secured and unsecured debt contracts are available for trade and analyzes this model to prove the existence and determine the nature of equilibria. I define a coexistence equilibrium in this economy as an equilibrium that involves active trade in both secured and unsecured debt, and study the conditions sufficient to guarantee its existence. This paper combines endogenous leverage with the anonymity of perfectly competitive markets to present a scenario where coexistence arises endogenously. I connect this behavior to the existence of endowment inequalities, and illustrate how this inequality affects agents' portfolio decisions between the two types of debt. Finally, by comparing equilibria across financial structures where only one or both kinds of contracts are available, I also demonstrate the asset pricing and redistributive implications of these results.

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1 Introduction

Writers and thinkers have often wondered why people repay their debts. Sometimes, they do it because they have to, and sometimes because they feel guilty if they choose not to. In general, however, people cannot always be trusted to repay what they borrow; the real fear of default that this engenders makes people hesitant to lend to others. For this reason, institutions were developed to enforce repayment. These institutions are usually punitive in nature – defaulters are punished in some form or the other. The implementation of these institutions can happen either ex post or ex ante. Consider, for example, the housing mortgage market; the mortgage contract states upfront that failing to repay the loan can lead to the seizure and repossession of the house being used as collateral – the use of collateral is thus an ex ante implementation of loan enforcement. On the other hand, in the case of sovereign debts, countries rarely borrow money after signing explicit agreements about what happens in the case of default; instead, if a country defaults on its debts, its creditors might choose ex post to impose some penalties on it. However, some markets, such as consumer and corporate finance, often involve both kinds of institutions – people may take a mortgage from a bank, but also use credit cards or borrow from a loan shark, and firms may have both secured and unsecured debt on their balance sheets.

How, then, do we account for the coexistence of secured and unsecured debt, or collateral and punishment? This paper aims to build a theoretical model and construct examples to study the existence, nature, and positive and normative consequences of default in a setting where both punishments and collateral simultaneously deter it. In other words, this amounts to modeling a scenario where secured and unsecured debt coexist, and default may be partial. To do this, I model a binomial economy where two kinds of assets exist – secured and unsecured. Agents are free to use either to borrow, and the aim is to understand under what circumstances agents make the decisions we observe them making in the real world.

First, we do have ample evidence that both kinds of debt do coexist, and in rather significant quantities, in both consumer and corporate finance[1].

Second, there is some evidence that richer households are less likely to hold unsecured debt and more likely to hold secured debt[2]. Among other results, Disney et al. (2010) observe, based on data from the British Household Panel Survey, that richer households (with greater holdings of financial assets) hold less unsecured debt. At the same time, the link between firm size/revenue and portfolio choice is an area of debate, with evidence in the literature going in both directions. He (2011) cites Frank and Goyal (2008) in arguing that smaller firms take on less leverage than larger firms. On the other hand, using a supervisory

[1]See Figure 1 for an illustration of this evidence using data from the New York Fed Consumer Credit panel; clearly, there are as many outstanding credit card accounts in the United States at any given point in the last few decades as all forms of secured debt combined, making unsecured debt a significant portion of consumer debt.

[2]See Figure 2 for a rough illustration of this, using data from the Survey of Consumer Finances.

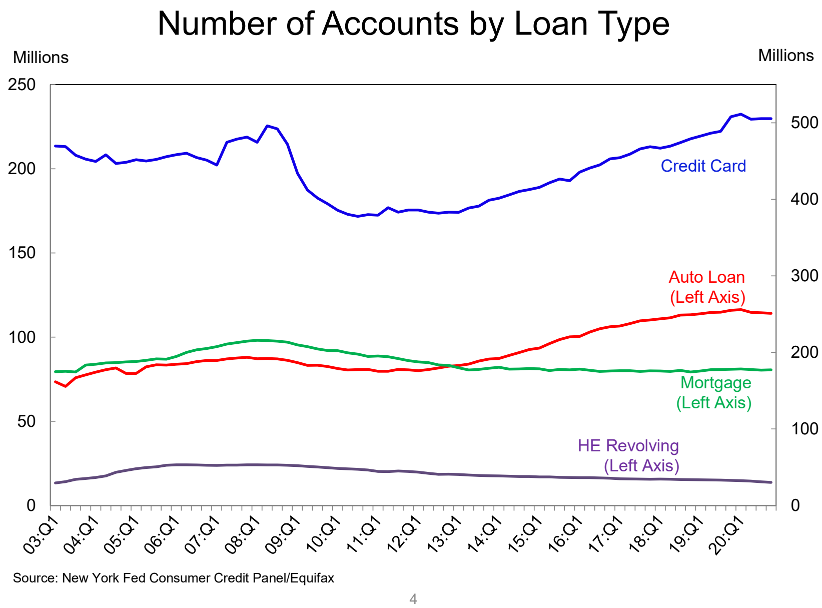

Figure 1: Coexistence of Debt Types

data set maintained by the Federal Reserve, Chodorow-Reich et al. (2022) report that smaller firms almost always post collateral, whereas larger ones often borrow unsecured.

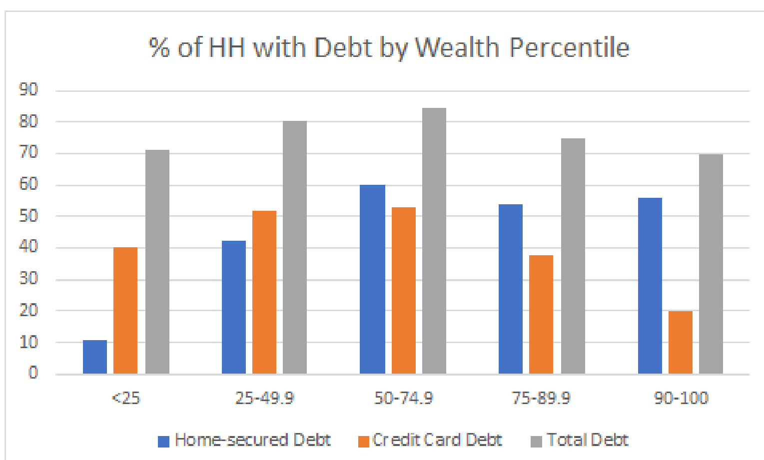

Figure 2: Debt Choice Over the Wealth Distribution

Third, both types of debt are known to affect asset pricing. Garriga et al. (2019); Garriga and Hedlund (2020) find credit to be an important factor in housing price dynamics. Landvoigt et al. (2015); Favilukis et al. (2017) find cheaper and easier access to credit, especially for poor households, was a major driver of the housing price boom in the 2000s. Justiniano et al. (2015) also find a close relationship between credit availability and housing prices, though they expect the relationship to be driven in the opposite direction. There is also some anecdotal evidence[3] that links student debt forgiveness and reduced borrowing requirements to the recent boom in housing prices. In the case of corporate finance, Scott (1977) is the seminal paper arguing that firm valuation can be incraesed by issuing secured debt. More recent work, such as Morellec (2001), finds that there is a more nuanced relationship between a firm's decision between secured and unsecured debt, and their valuation.

As such, I aim to build a perfectly competitive GE model that can endogenously explain the coexistence of secured and unsecured debt. Such a model will allow me to study both how inequality (in the form of endowment heterogeneity) affects the portfolio choice between debt types, and the effect of such coexistence on asset prices.

I model the secured part of the debt market after the literature on endogenous leverage; a financial contract in this economy consists not just of the promise it makes, but also the collateral used to back it. The necessity of collateral to secure borrowing both limits the amount that can be borrowed (and hence defaulted on) and acts as a deterrent against extreme strategic default – where the agent defaults despite the value of the collateral held by them being sufficient to repay the loan. Dubey et al. (1995); Geanakoplos (1997); Geanakoplos and Zame (1997, 2002) were among the first papers to present the C-model (or collateral GEI model), where financial promises need to be backed by collateral requirements. Papers like Geanakoplos (2003); Fostel and Geanakoplos (2008, 2015) built on these ideas and developed the concepts of leverage cycles and collateral and liquidity values. These papers demonstrate how collateral requirements have profound positive and normative implications for the economy.

The unsecured part of the debt market in my model is built on another strand of the literature on default, epitomized by Dubey et al. (2005), considers another tool that can serve a similar purpose – punishment; a financial contract in this economy (also called the λ-model) consists of promises, punishments, and borrowing constraints. By using a "pangs of conscience" punishment that is increasing in the magnitude of the default, they are able to show that markets can function in an orderly fashion even in the presence of default, and that punishment and borrowing constraints can provide generic existence of equilibria in a GEI setting with default.

I seek to bring together these two strands of literature by asking similar questions while combining both methods of disciplining default. In a real-world scenario, borrowing from a bank using loans that are secured using collateral can be seen as an example of secured debt. On the other hand, borrowing with punishment-on-default can be seen as borrowing from loan sharks; they might not ask for collateral, and it might be

[3]See https://www.wsj.com/articles/the-student-debt-bubble-fueled-a-housing-bubble-debt-income-obama-fannie-freddie-bd29b05c .

possible to default partially, but they back their lending with threats of punishment (often physical) in case of default. The key mechanism of interest here concerns the endogenous selection of the contracts that agents trade actively and the determination of the credit surface and equilibrium leverage. These questions cannot be studied using the two kinds of contracts separately – the selected contract and the equilibrium price (interest rate) of each type are likely to be affected by each other's existence. At the same time, we may also question whether an equilibrium of a model that includes both asset classes will necessarily involve trade in both. Is it possible that the agents choose to use either secured or unsecured credit and not the other? Under what features of the model do both asset classes "coexist" in equilibrium?

To the best of my knowledge, such a model of both kinds of debt in GE with incomplete, perfectly competitive markets does not currently exist in the literature. Existing models either focus on the case of complete markets (Araujo and Villalba, 2022) or rely on partial equilibrium or other frameworks (Athreya, 2006; Donaldson et al., 2020), making my model a novel contribution.

My analysis is relevant for a few reasons: first, it brings together two strands of literature that both deal with how default is deterred, thereby explaining the coexistence of secured and unsecured debt. In and of itself, this is a non-trivial task, given the additional complexity engendered by putting two already complex models together. Second, these questions have real-world significance: most debtor credit markets in the real world have a mixture of secured and unsecured debt. Thus, this project may be a step towards an analysis of default in such generalized settings. Third, the existence of unsecured (punishment-enforced) debt is likely to have spillover effects on the market for collateral-backed (secured) debt, and vice versa, which are likely to be crucial to explaining the kinds of questions this model can answer regarding portfolio decisions, asset pricing, or the spillover effects of government regulation in either debt market on the other. These interactions imply that there is real value in modeling both forms of debt together rather than in isolation.

To expand upon that last point, this paper fits into an agenda that explores how financial innovation affects pre-existing asset markets. In another working paper (Fostel et al., 2023), we explore the effect of financial innovation within the secured debt market on the price of assets used to back the secured debt. On the other hand, this paper demonstrates the effect of financial innovation introducing a new debt market (secured or unsecured) on the price of the asset used to back the secured debt contracts. This model, once developed further, can serve as a new starting point for this agenda, and may be helpful in answering other questions, such as how policy aimed at one debt market might spill over into the other. For example, one of my upcoming research goals is to to study how mortgage subsidies (a policy aimed at the secured debt market) affect agents' actions in the unsecured debt market.

This model synthesizes two complex strands of literature, each focused on one of the mechanisms that encourage debt repayment, to shed light on the coexistence of secured and unsecured debt —- a phenomenon that is not only theoretically intriguing but also empirically significant. The task of integrating these two models is far from trivial and presents its own set of challenges. However, the endeavor is well-justified given the real-world relevance of the questions at hand. Both secured and unsecured debt are pervasive and coexist in substantial proportions within consumer and corporate finance markets. Therefore, this project serves as a foundational step toward a more comprehensive analysis of default mechanisms in such generalized financial settings.

Moreover, the model offers qualitative insights into the portfolio decisions of agents across different wealth brackets. Specifically, it suggests that wealthier agents are more inclined to hold a greater proportion of secured debt, while reducing their exposure to unsecured debt. This observation has important implications for understanding financial behavior across socio-economic strata.

The rest of the paper is structured as follows. In Section 2, I set up a binomial two-period general equilibrium model, where agents have access to two goods – a numeraire consumption good and a perfectly durable non-financial asset – and menus of two kinds of debt. Secured debt is backed by the ex ante use of collateral, and each secured debt contract is characterized by the promise of repayment in units of the numeraire good, and the amount of collateral used to back that promise. Unsecured debt is backed by an ex-post scaling utility penalty in the case of default, and each unsecured debt pool is characterized by the promise of repayment in terms of the numeraire good, the penalty parameter, and the sales cap. Unsecured debt is characterized as pools because I follow Dubey et al. (2005) in modeling unsecured debt as being intermediated by pools that collect repayment from agents as a measure of retaining anonymity while allowing default to be punished. I proceed to prove the existence of equilibria in this λC-economy under standard assumptions, thereby guaranteeing that the model is internally consistent.

Next, in Section 3, I define a coexistence equilibrium in this economy as an equilibrium that involves active trade in both secured and unsecured debt. This definition allows me to identify sufficient conditions under which all equilibria of this model must feature coexistence. This relies on the ideas that secured debt offers better returns, and hence, all agents would prefer to first take out as much secured debt as they can, while in the presence of sufficient endowment heterogeneity, at least one agent would also want to take on unsecured debt as well, in order to facilitate greater access to secured debt.

Then, in Section 4, I present a simple numerical example to illustrate the features of the model, and use this example to demonstrate other features of the equilibria I am interested in. In particular, I construct an example in which richer agents hold more secured debt, and less unsecured debt, a pattern that is reflected in real world data. I use this example to present further sufficient conditions (in addition to those sufficient to guarantee coexistence) under which we obtain a coexistence equilibrium of the λC-economy that displays this property.

Finally, in Section 5, I use the same numerical example to compare the price of the non-financial asset across various economies that differ only on the basis of what financial markets are open to agents for trade. To be specific, I compare the equilibrium in the complete λC-economy to equilibria in models that are identical except that agents only have access to either secured debt (C-economy) or unsecured debt (λ-economy), but not both. This comparison tells us that, in my constructed example, moving from either of the single-debt-type economies to the λC-economy pushes up the price of the non-financial asset. This can be interpreted as an effect of financial innovation, if an indirect one in the case of moving from the C-economy to the λC-economy. Section 6 concludes and presents implications for future work.

2 Model

I use the standard binomial two-period general equilibrium model, with two time periods, $t \in \{0, 1\}$, with two states in the second time period such that the state space is $S \equiv \{0, U, D\}$, with the set of terminal states being given by $S' \equiv \{U, D\}$. Let there be one consumption good c that we treat as the numeraire, and one perfectly durable non-financial asset y[4]. Denote the price of the asset y in state s by p_s.

Let there be a continuum of agents, $h \in H$, characterized by their subjective discount factors (β^h) and probabilities (γ_s^h), utility functions (u^h), and endowments ($e^h \equiv \left(\left\{e_s^h\right\}_{s\in S}, y^h\right)$)[5] such that their expected utility is given by

$$U^h = u^h\left(c_0^h, y_0^h\right) + \beta^h \sum_{s\in S'} \gamma_s^h u^h\left(c_s^h, y_s^h\right).$$

We make the following standard assumptions about the utility functions and endowments of the agents:

Assumption A1. *Everybody owns something in every state:* $e_s^h \neq 0, \forall h \in H, s \in S$.

Assumption A2. $\forall h \in H, u^h(\cdot)$ *is weakly concave in each of its arguments.*

Assumption A3. $\forall h \in H, u^h(\cdot)$ *is weakly monotone in each of its arguments.*

Assumption A4. $\forall h \in H, u^h(\cdot)$ *is continuously differentiable in each of its arguments.*

The crux of our model is the existence of menus of two kinds of debt contracts - secured and unsecured. The menu of secured debt contracts is modeled in the vein of the endogenous leverage literature, as in Geanakoplos (1997); Fostel and Geanakoplos (2008, 2015), defined as $(j \cdot \tilde{1}, 1) \in J$, where each secured debt contract j is characterized by the promise of repayment of j units of consumption made against a

[4]In particular, I assume that the asset y directly provides utility, thereby making it a non-financial asset.

[5]Notice the inherent assumption that agents are endowed with the financial asset y only at time 0; no additional endowments of y are realized at time 1 in either state of the world.

collateral of one unit of the asset y. Agents choose to buy or sell as many and whichever contracts they want, taking prices as given, with the result that the choice of leverage is endogenous. Following the standard structure in the endogenous collateral literature, we know that the per-contract delivery to creditors is given by $\delta_{sj} = \min\{j, p_s\}$[6]. We define the price (amount borrowed) of the contract j by π_j.

The menu of unsecured debt contracts is modeled in the vein of the literature on punishment, as in Dubey et al. (2005), defined as $(R_i \cdot \tilde{1}, \lambda_i, Q_i) \in I$, where each unsecured debt contract or "pool" i is defined by the promise of repayment of R_i units of consumption made under a threat of utility penalty scaled by the factor λ_i and sales caps Q_i. Agents choose to buy or sell as many and whichever contracts they want, taking prices as given, with the result that the choice of pools is endogenous. Following the structure of unsecured debt contracts in Dubey et al. (2005), delivering D_i instead of R_i incurs a penalty of $\lambda_i \max\{R_i - D_i, 0\}$. Note that the penalty parameter λ_i depends only on the chosen pool, and not the person borrowing using the pool. We define π_i as the price of contract i and D^h_{si} as the repayment being made against the unsecured debt contract i in terminal state s by agent h. Repayments made against each unsecured debt contract by agents are pooled before being repaid pro rata to creditors[7], such that the per-contract delivery to creditors can be defined as

$$\delta_{si} = R_i \frac{\sum_h D^h_{si}}{\sum_h R_i \varphi^h_i} = \frac{\sum_h D^h_{si}}{\sum_h \varphi^h_i},$$

implying that the repayment rate on unsecured debt contract i in state s is given by $K_{si} = \frac{\delta_{si}}{R_i}$.

2.1 Economy

Based on the above definitions of the states, goods, agents, and debt contracts, we can define the economy of our model as follows:

Definition 1. *Given the state space S, the agents $h \in H$ defined by their endowments e^h and utilities u^h, and the menus of secured (J) and unsecured (I) debt contracts, and under Assumptions A1-A4, we define the economy we are working in as the λC-economy, $E_{\lambda C}$, as given by*

$$E_{\lambda C} = \left(S, \left(u^h, e^h\right)_{h\in H}, J, I\right).$$

We can also further define a pair of special cases of the economy as follows:

Definition 1a. *When only unsecured debt contracts are available for trade, i.e., $J = \varnothing$, the λC-economy*

[6] This is easily seen from the fact that no agent would repay more on a contract in any given state than what the collateral backing that contract was worth in that state.

[7] The concept of pooling debt here is somewhat similar to the concept of asset markets with heterogenous quality and adverse selection used in Guerrieri and Shimer (2014).

reduces to the special case of the λ-economy,

$$E_\lambda = \left(S, \left(u^h, e^h\right)_{h\in H}, \varnothing, I\right).$$

Definition 1b. *When only secured debt contracts are available for trade, i.e., $I = \varnothing$, the λC-economy reduces to the special case of the C-economy,*

$$E_C = \left(S, \left(u^h, e^h\right)_{h\in H}, J, \varnothing\right).$$

2.2 Budget Set

Given the prices of goods and debt contracts as well as the expected repayment rates of unsecured debt contracts, agents choose consumption and holdings of whatever debt contracts of either or both types as they want to maximize post-penalty expected utility

$$W^h = U^h - \sum_i \lambda_i \sum_s \gamma_s^h \left[\varphi_i^h R_i - D_{si}^h\right]^+$$

subject to the budget set

$$\begin{aligned}
B^h(p, \pi_j, \pi_i, K_{si}) = \Big\{&(c, y, \theta_j, \theta_i, \varphi_j, \varphi_i, D_{sj}^h, D_{si}^h) : \\
&\left(c_0^h - e_0^h\right) + p\left(y_0^h - y^h\right) + \sum_j \pi_j\left(\theta_j^h - \varphi_j^h\right) + \pi_i\left(\theta_i^h - \varphi_i^h\right) \leq 0; \\
&\left(c_s^h - e_s^h\right) + p_s\left(y_s^h - y_0^h\right) + \sum_i D_{si}^h + \sum_j \varphi_j^h \min\{j, p_s\} - \sum_i \theta_i^h K_{si} R_i - \sum_j \theta_j^h \min\{j, p_s\} \leq 0, \forall s \in \{U, D\} \\
&\sum_j \max\{0, \varphi_j^h\} \leq y_0^h\Big\}
\end{aligned}$$

2.3 Equilibrium

Having defined the environment of the model that I am working in, I will now proceed to define the solution concept I will be using.

Definition 2. *A collateral-punishment (λC) equilibrium for this economy is defined as a vector comprising of prices (prices for the asset and financial contracts and expected deliveries on unsecured debt) and allocations (individual consumptions of the numeraire good and the asset, sales and purchases of both kinds of assets, and actual deliveries for both kinds of assets),* $\left(p, (\pi_j)_j, (\pi_i, K_i)_i, \left(c^h, y^h, \left(\theta_j^h, \varphi_j^h, \left(D_{sj}^h\right)_s\right)_j, \left(\theta_i^h, \varphi_i^h, \left(D_{si}^h\right)_s\right)_i\right)_h\right)$, *such that*

1. *the allocations solve the agents' maximization problems*

$$\left(c^h, y^h, \left(\theta_j^h, \varphi_j^h, \left(D_{sj}^h\right)_s\right)_j, \left(\theta_i^h, \varphi_i^h, \left(D_{si}^h\right)_s\right)_i\right) \in \arg\max W^h \left(c^h, y^h, \left(\theta_j^h, \varphi_j^h, \left(D_{sj}^h\right)_s\right)_j, \left(\theta_i^h, \varphi_i^h, \left(D_{si}^h\right)_s\right)_i, p\right)$$

over their budget set $B^h(p, \pi_j, \pi_i, K_i), \forall h$,

2. *the market for the numeraire clears in all states*

$$\sum_{h \in H} \left(c_0^h - e_0^h\right) = 0, \sum_{h \in H} \left(c_s^h - e_s^h\right) = \sum_{h \in H} (y_0^h - y_s^h) p_s, s \in S',$$

3. *the markets for the collateral asset and financial contracts of both types clears at* $t = 0$

$$\sum_{h \in H} \left(y_0^h - y^h\right) = 0, \sum_{h \in H} \left(\theta_i^h - \varphi_i^h\right) = \sum_{h \in H} \left(\theta_j^h - \varphi_j^h\right) = 0, \forall i, j, \textit{ and}$$

4. *lenders form rational expectations of the delivery from any unsecured debt contracts that are actually traded in equilibrium*

$$K_{si} = \begin{cases} \frac{\sum_{h \in H} p_s D_{si}^h}{\sum_{h \in H} p_s R_i \varphi_i^h} = \frac{\delta_{si}}{R_i}, & \sum_{h \in H} p_s R_i \varphi_i^h > 0 \\ \textit{arbitrary}, & \sum_{h \in H} p_s R_i \varphi_i^h = 0 \end{cases}, \forall i.$$

This general notion of an equilibrium is problematic in this context, as discussed in Dubey et al. (2005); the fact that expectations of delivery are arbitrary for unsecured debt contracts that are not actively traded in equilibrium leads to the possibility that some contracts may be go untraded simply due to what they call "whimsical pessimism" - a situation where agents arbitrarily assume that a particular contract will always be defaulted upon, resulting in it not being traded, which in turn allows the original arbitrary assumption. In order to avoid such arbitrary exclusions of certain contracts, we need to refine our equilibrium concept to account for off-equilibrium behavior. We do so by following the procedure of ϵ-boosting as described in Dubey et al. (2005).

An ϵ-boosted economy is defined as a perturbation of the economy described above where we introduce an infinitesimal agent who borrows an infinitesimal amount ϵ using each unsecured debt contract available on the menu, and always fully repays any debt he/she takes out.

Definition 2a. *A collateral-punishment equilibrium of an* ϵ*-boosted economy is known as an* ϵ*-boosted collateral-punishment* ($\epsilon\lambda C$) *equilibrium.*

Definition 3. *A λC-equilibrium $\mathcal{E}_{\lambda C}$ is called a refined equilibrium if there exists a sequence of ϵ-boosted collateral-punishment ($\epsilon\lambda C$) equilibria $\mathcal{E}(\epsilon)$ s.t.* $\lim_{\epsilon\to 0}\mathcal{E}(\epsilon) = \mathcal{E}_{\lambda C}$.

Being the limit of a sequence of ϵ-boosted collateral-punishment ($\epsilon\lambda C$) equilibria which do not feature "whimsical pessimism", we cn be assured that refined equilibria are also free of this problem, and can therefore be considered the core solution concept of this model.

2.4 Existence

Proposition 1. *Consider the λC-economy $E_{\lambda C}$; then, a refined equilibrium exists.*

Proof. See Appendix A.1. □

The core of this proof depends on using a fixed point theorem on a mapping from a non-empty, compact, and convex space to ensure the existence of a fixed point that can serve as an equilibrium. However, given the large number of secured and unsecured debt contracts that agents have access to, their ability to default, as well as severe market incompleteness, it is quite easy to be in a position where the conditions necessary for the use of a fixed point theorem may not apply. However, under the fairly standard assumptions described earlier, I am able to combine the methods that Fostel and Geanakoplos (2015) and Dubey et al. (2005) use in the case of the standard collateral and punishment models respectively to restore these conditions, and adapt them to work in the case where both kinds of debt contracts coexist. This allows me to prove that a refined equilibrium exists in this economy.

3 Coexistence

Coexistence is defined as the existence of trade in at least one contract of each type, i.e. $\exists i \in I, j \in J$ such that $\sum\limits_{h\in H}\theta_j^h > 0$ and $\sum\limits_{h\in H}\theta_i^h > 0$. A refined equilibrium that satisfies the property of coexistence is called a refined coexistence equilibrium. Represent the property of coexistence by ω, and the set of all such refined equilibria by $\mathcal{E}(\omega)$. The primary question we ask in this section is under what conditions this set is non-empty, i.e., $\mathcal{E}(\omega) \neq \phi$.

Definition 4. *A refined competitive equilibrium is a refined coexistence equilibrium if there is trade in at least one contract of each type, i.e. $\exists i \in I, j \in J, h, h' \in H$ such that $\theta_j^h > 0$ and $\theta_i^{h'} > 0$.*

In order to prove the coexistence of both kinds of debt, we consider a λC-economy as described in previous sections with a few additional assumptions:

Assumption C1. *Agents can be divided into two groups, i.e. $H \equiv H^L \bigcup H^B$, such that*

1. *The utility function of agents is such that agents of type B always like the asset more than agents of type L,*

$$\forall h \in H^L, h' \in H^B, u_Y^h\big|_{Y^h=0} < u_Y^{h'}\Big|_{Y^{h'}=\sum\limits_{h\in H} y^h}.$$

2. *Furthermore, assume that agents of type L are risk-neutral.*

Assumption C2. *Agents' endowments are such that*

1. *At least one agent of Type L is endowed with some of the asset Y at time 0, i.e.,*

$$\exists h \in H^L \text{ s.t. } y^h \neq 0.$$

2. *All agents of type B need to borrow; the poorest agent is unable to afford the down payment, i.e.,*

$$\min_{h'\in H^B} e_0^{h'} = 0 \text{ and } \max_{h'\in H^B} e_0^{h'} < \bar{e} \text{ for some finite } \bar{e}$$

3. *Endowments in the bad state are bounded away from zero, i.e., $\exists \epsilon > 0$ s.t.$\forall h \in H^B$,*

$$e_{Dc}^h > \epsilon > yp_D + c_D^h$$

We begin first by proving an intermediate result.

Lemma 1. *Consider a λC-economy $E_{\lambda C}$ satisfying Assumptions C1-C2. Then, given any competitive equilibrium where secured debt contracts are being traded actively, no secured debt contract that is being actively traded can offer 100% LTV.*

Proof. In a secured-debt-only equilibrium, the MU of using cash to buy the asset Y and consumption c at time 0 must be equal:

$$\frac{U_y^h(c_0^h, y_0^h) + \sum\limits_{s\in S'} \mu_s^h\,(p_s - \delta_s(j))}{p_0 - \pi_j} = \frac{U_c^h(c_0^h, y_0^h)}{1}.$$

Since $U_y^h(c_0^h, y_0^h) \neq 0$ for a non-financial asset, $p_0 \neq \pi_j \implies LTV \neq 100\%$. □

Proposition 2. *Consider a λC-economy $E_{\lambda C}$ satisfying Assumptions C1-C2. Then, any refined competitive equilibrium is a refined coexistence equilibrium.*

Proof. See Appendix A.2. □

While relegating the minutiae of this proof to the appendix, I will provide here a sketch of the steps taken to prove this proposition. First, we know that an equilibrium exists, as per Proposition 1. Consider any such equilibrium. We also know, from Assumptions C1 and C2.1, that this equilibrium must feature trade in goods. We further know from Assumption C2.2 that this equilibrium must feature trade in financial contracts. Given this information, the next question is whether the equilibrium could feature trade in only one kind of debt contract.

Assume that the equilibrium features trade only in secured debt contracts. Since, per Lemma 1, no secured debt contract that is being traded actively can offer 100% LTV, any purchase of the asset y must require a down payment. However, by Assumption C2.2, the poorest agent is unable to afford the down payment by using only their endowment. Hence, this agent will unilaterally deviate to using unsecured debt contracts in order to afford the down payment necessary to access secured debt contracts. This provides a contradiction.

Now assume that the equilibrium features trade only in unsecured debt contracts. I prove that, given Assumptions C2.2 and C2.3, there is a feasible and profitable deviation for some agent, taking prices as given. This comes down to the idea that, for the same amount borrowed (or promised), secured debt offers better returns, and would be the preferred choice for any agent who can afford it. In other words, it is wasteful to not leverage collateral that you have access to. This provides a contradiction.

Since any given equilibrium cannot feature trade in only one kind of debt contract, it must feature coexistence.

4 Debt Portfolio Composition

Next, I consider a simple numerical example to illustrate the results we have discussed so far. I will then use this example to further study some additional results.

4.1 Numerical Example

Consider a special case of the λC-economy, under Assumptions C1-C2, with some additional structure imposed to facilitate further analysis. Assume that there are two groups of agents, $H \equiv H^L \bigcup H^B$, such that their state utility functions are given by

$$u^L = y^L + c^L$$
$$u^B(h) = \sqrt{y^B(h)} + \alpha\sqrt{c^B(h)},$$

which clearly meet the requirements of Assumptions A2-A4 and C1. We also abstract away, for the sake of simplicity, from considering discounting across time, since that is not a particular focus of this analysis; we assume that $\beta = 1$ for all agents. Assume further that the endowments of the two groups of agents are given by

$$e^L = ((20, 1), (20, 0), (20, 0))$$
$$e^B(h) = ((8h, 0), (20, 0), (3, 0)), \forall h \in (0, 1),$$

which is in line with the requirements of assumptions A1 and C2. Further assume that agents have access to both secured and unsecured debt markets, with the menu of available debt contracts/pools being given by

$$I = \left\{(d_i.\tilde{1}, \lambda_i, \infty; \lambda_i \in \left[\frac{1}{2}, 10\right]\right\}$$
$$J = \{(j.\tilde{1}, 1)_{j\in\mathbb{R}_+}\}.$$

I also assume that $\epsilon = 0.1$, i.e., that there is a 10% chance of ending up in the bad state of the world, D, at time 1.

Some of these additional assumptions have been chosen such that we obtain an equilibrium where, as we will see, the only secured debt contract being actively traded is one that promises $j^* = p^U$, and the only unsecured debt pools being used are ones with $\lambda^* = \frac{1}{2}$. Furthermore, all traded secured debt contracts deliver fully in the good state U but are defaulted on in the bad state, D. On the other hand, all traded unsecured debt pools deliver fully in the good state U, and exhibit partial default in the bad state D.

Any secured debt contract that promises $j \leq p^D$ will never be defaulted on if it is issued; this means that such debt contracts will always be charged a riskless rate of interest, $R_j = 1$. Then, no borrower will ever sell a contract with $j < p^D$, since the opportunity cost of collateral in doing so is forgoing the ability to borrow using $j = p^D$ at the same rate of interest. Any secured debt contract that promises $j > p^U$ will always be defaulted on if it is issued; this means that no borrower will ever sell a contract with $j > p^U$. Restricting our focus to $j \in [p^D, p^U]$, agents face a trade-off between a higher ability to borrow and the higher interest rates they will have to face in order to do so. Given that all agents are collateral-constrained (and hence borrowing-constrained) at time 0, and that they expect sufficiently higher endowments in the future (both of which conditions hold under the example I have constructed), they will choose to pay the higher interest rate to borrow as much as they can, thereby using only the contract $j^* = p^U$.

In the case of unsecured debt pools, the actual repayment in the bad state is monotonically increasing in the choice of λ; a pool with a lower penalty parameter does not present as large an incentive for repayment, and agents choose to consume more of their endowment and repay less of their promise in state D, resulting in a higher interest rate and a lower amount borrowed. However, the parameters in this example are chosen such that the benefits of choosing a higher penalty parameter in equilibrium, both in terms of the direct benefit of a lower interest rate and the indirect benefit of a relaxed time-0 budget constraint, are lower than the incurred cost of the increased penalty faced when they default in state D.

The range of λ that agents can choose from for unsecured debt pools are chosen such that we observe the type of default behavior mentioned above. The lower limit on the penalty parameter is tailored to incentivize the agents' behavior; it is not so high as to induce agents to spend all their endowment trying to repay what they can of the debt, but not so low as to encourage complete default in state D or any kind of default in state U. At the same time, the upper limit on λ is such that agents do not benefit so much from choosing higher penalties as to deviate away from choosing the lowest penalty available, as described above.

In such an equilibrium, agents choose to borrow using unsecured debt until the additional cost tomorrow of borrowing an extra unit of consumption today equals the marginal benefit today of that extra unit of consumption, or the marginal benefit of using that unit of consumption as down payment to leverage an additional amount of housing, i.e.,

$$\frac{\alpha}{2\sqrt{c_0^h}} = \frac{1}{2(p_0 - \pi_j)\sqrt{y_0^h}} = \frac{\alpha}{2\sqrt{c_U^h}} R_i^h (1 - \epsilon) + R_i^h \underline{\lambda} \epsilon$$

Note that leveraging the house provides benefits both in terms of the utility flow it provides today as well as the consumption it can be sold for tomorrow; however, the leverage contract also means that while agents only pay a down payment (an not full price) for the house today, they also lose a claim to (at least) part of the value of the house tomorrow, depending on the secured debt contract that they choose. In this equilibrium, where the debt contract chosen is $j^* = p^U$, agents lose the value of the entire house tomorrow, and are effectively using the down payment to pay for the utility flow received today.

Agents also further choose food and housing allocations to spend their constrained budget in an optimal way, both in terms of intra-temporal trade-offs between food and housing, and inter-temporal trade-offs between the states 0, U, and D. This results in the following set of equations that pin down the equilibrium.

$$\frac{\alpha}{2\sqrt{c_D^h}} = \lambda$$

$$y_D^h = 1$$

$$\frac{\alpha}{2\sqrt{c_D^h}} = \frac{1}{2p_D\sqrt{y_D^h}}$$

$$r_D^h + c_D^h + p_D = e_D^h$$

$$\int_0^1 y_U^h dh = 1$$

$$p_U y_U^h + c_U^h + d_U^h = e_U$$

$$\frac{\alpha}{2\sqrt{c_U^h}} = \frac{1}{2p_U\sqrt{y_U^h}}$$

$$\pi_i^h = d_U^h(1-\epsilon) + r_D^h\epsilon$$

$$R_i^h = \frac{d_U^h}{\pi_i^h}$$

$$\int_0^1 y_0^h dh = 1$$

$$(p_0 - \pi_j)y_0^h + c_0^h = 8h + \pi_i^h$$

$$\frac{\alpha}{2\sqrt{c_0^h}} = \frac{1}{2(p_0 - \pi_j)\sqrt{y_0^h}}$$

$$\pi_j = p_U(1-\epsilon) + p_D\epsilon$$

$$\frac{\alpha}{2\sqrt{c_0^h}} = \frac{\alpha}{2\sqrt{c_U^h}} R_i^h(1-\epsilon) + R_i^h\underline{\lambda}\epsilon$$

4.2 Portfolio Composition

In equilibrium, all agents of type B borrow using both secured debt contracts and unsecured debt pools. Moreover, richer (in terms of time-0 endowments) agents hold more secured and less unsecured debt (Figure 3). Since all agents borrow using the same secured debt contracts, this translates into richer agents purchasing (and hence leveraging) a larger stock of housing at time 0. Conditional on an internal solution to agent optimization (which has to be the case based on the assumptions of this model), this translates into a larger consumption of food at time 0. Finally, based on agent optimization and the agents' budget constraints, this also implies that richer agents take on less unsecured debt. Agents with different initial (time-0) endowments borrow different amounts using different unsecured debt pools; although all pools in use have the same $\lambda = \frac{1}{2}$, they promise different amounts of consumption at time 1, and hence cost (allow agents to borrow) different amounts today. Furthermore, we see that richer agents are borrowing (less) at better terms, i.e., they face lower interest rates.

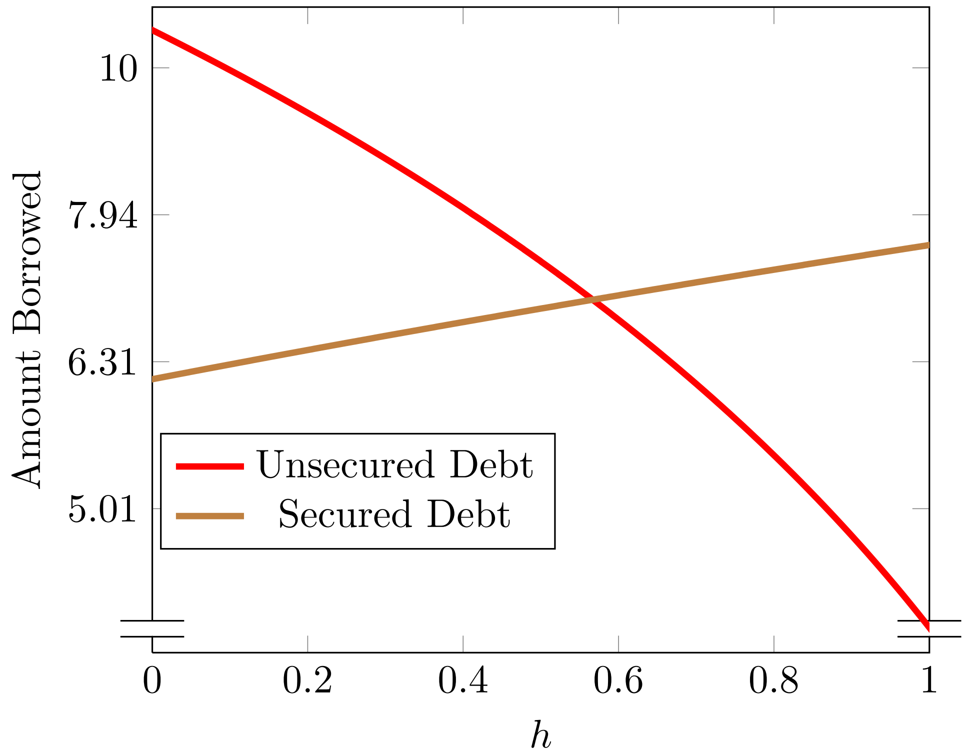

Figure 3: Portfolio Choice Over the Endowment Distribution

Now, let us consider under what sufficient conditions this result holds more generally. The starting point is to consider what conditions imposed in this example led to the regime described above. Two primary requirements come up in this context. First, we require the range of penalty parameters to be such that a regime of full repayment in state U and partial and interior repayment in state D is supported. Second, the state utility function for borrowers must be such that intra-temporal optimization has an interior solution in food and housing. This can be boiled down to the following additional assumptions.

Assumption P1. *The utility penalty parameter for all available unsecured debt contracts is bounded both above and below, i.e.,* $\lambda_i \in [\underline{\lambda}, \bar{\lambda}]$ *,* $\forall i \in I$*, s.t.* $\underline{\lambda}$ *is not so low as to induce strategic default in state* U *and* $\bar{\lambda}$ *is not so high as to induce repayment being the only expenditure in the bad state if such a contract is chosen.*

Assumption P2. *The state utility is given by* $u(c, y) = v(y) + w(c)$*, where* $w(\cdot)$ *is more concave than* $v(\cdot)$*.*

Under these assumptions, I can now make a statement regarding agents' portfolio choice.

Proposition 3. *Consider a* λC*-economy* $E_{\lambda C}$ *satisfying Assumptions C1-C2 and P1-P2. Then,*

- *secured debt,* $|\pi_j y_j^h|$*, is increasing in* h*, and*

- *unsecured debt,* $|\pi_i^h|$*, is decreasing in* h*.*

Proof. See Appendix A.3. $\square$

Secured debt provides better returns and is the preferred choice when feasible. As I argued in the sketch of the coexistence proof, it is wasteful to not leverage collateral that you have access to. As agents get richer, they substitute away from unsecured debt and into secured debt, as they become capable of purchasing and leveraging larger stocks of housing. Internal optimization then implies that these richer agents must be consuming more food, both today and tomorrow, thereby implying that they must be borrowing (and hence repaying) less using unsecured debt pools.

5 Asset Pricing

Consider once again the numerical example from earlier. I will first present two variants of this economy, and compare equilibria across the three economies. The first variant is one where $J = \varnothing$ but $I = \left\{(d_i.\tilde{1}, \lambda_i; \lambda_i \in \left[\frac{1}{2}, 10\right]\right\}$ as in the original λC-economy; agents have access to the same menu of unsecured debt pools, but have no access to secured debt contracts. Since agents in this economy have access to unsecured debt, but not secured debt, I call this economy a λ-economy. The second variant is one where $I = \varnothing$ but $J = \{(j.\tilde{1}, 1)_{j \in \mathbb{R}_+}\}$ as in the original λC-economy; agents have access to the same menu of secured debt contracts, but have no access to unsecured debt pools. Since agents in this economy have access to secured debt, but not unsecured debt, I call this economy a C-economy. Both these economies are identical to the λC-economy in all other parameters except the debt markets they have access to.

In the λ-economy, the only unsecured debt pools being used are ones with $\lambda^* = \frac{1}{2}$. Furthermore, all traded unsecured debt pools deliver fully in the good state U, and exhibit partial default in the bad state D. In the C-economy, the only secured debt contract being actively traded is one that promises $j^* = p^U$, and it delivers fully in the good state U but is defaulted on in the bad state, D.

In the λ-equilibrium, agents once again choose to borrow using unsecured debt until the additional cost tomorrow of borrowing an extra unit of consumption today equals the marginal benefit today of that extra unit of consumption, i.e.,

$$\frac{\alpha}{2\sqrt{c_0^h}} = \frac{\alpha}{2\sqrt{c_U^h}} R_i^h(1-\epsilon) + R_i^h \underline{\lambda}\epsilon$$

Agents also further choose food and housing allocations to spend their constrained budget in an optimal way, both in terms of intra-temporal trade-offs between food and housing, and inter-temporal trade-offs between the states 0, U, and D. This results in the following set of equations that pin down the equilibrium.

$$
\begin{aligned}
\frac{\alpha}{2\sqrt{c_D^h}} &= \underline{\lambda} \\
\int_0^1 y_D^h &= 1 \\
\frac{\alpha}{2\sqrt{c_D^h}} &= \frac{1}{2p_D\sqrt{y_D^h}} \\
r_D^h + c_D^h + p_D y_D^h &= e_D^h \\
\int_0^1 y_U^h dh &= 1 \\
p_U y_U^h + c_U^h + d_U^h &= e_U + p_U y_0^h \\
\frac{\alpha}{2\sqrt{c_U^h}} &= \frac{1}{2p_U\sqrt{y_U^h}} \\
\pi_i^h &= d_U^h(1-\epsilon) + r_D^h\epsilon \\
R_i^h &= \frac{d_U^h}{\pi_i^h} \\
\int_0^1 y_0^h dh &= 1 \\
p_0 y_0^h + c_0^h &= 8h + \pi_i^h \\
\frac{\alpha}{2\sqrt{c_0^h}} &= \frac{1}{2p_0\sqrt{y_0^h}} + \frac{(1-\epsilon)p_U\frac{\alpha}{2\sqrt{c_U^h}} + \epsilon p_D\frac{\alpha}{2\sqrt{c_D^h}}}{p_0} \\
\frac{\alpha}{2\sqrt{c_0^h}} &= \frac{\alpha}{2\sqrt{c_U^h}}R_i^h(1-\epsilon) + R_i^h\underline{\lambda}\epsilon
\end{aligned}
$$

In a C-equilibrium, agents are restricted in terms of borrowing using secured debt by the stock of collateral owned by them. Agents also further choose food and housing allocations to spend their constrained budget in an optimal way, both in terms of intra-temporal trade-offs between food and housing, and inter-temporal trade-offs between the states 0, U, and D. This results in the following set of equations that pin down the equilibrium.

$$y_D^h = 1$$
$$\frac{\alpha}{2\sqrt{c_D^h}} = \frac{1}{2p_D\sqrt{y_D^h}}$$
$$c_D^h + p_D = e_D^h$$
$$\int_0^1 y_U^h dh = 1$$
$$p_U y_U^h + c_U^h = e_U$$
$$\frac{\alpha}{2\sqrt{c_U^h}} = \frac{1}{2p_U\sqrt{y_U^h}} \quad \int_0^1 y_0^h dh \qquad = 1$$
$$(p_0 - \pi_j)y_0^h + c_0^h = 8h$$
$$\frac{\alpha}{2\sqrt{c_0^h}} = \frac{1}{2(p_0-\pi_j)\sqrt{y_0^h}}$$
$$\pi_j = p_U(1-\epsilon) + p_D\epsilon$$

Solving for both these equilibria as well, and comparing the solutions to the λC-equilibrium, we observe that the price of the house at time 0 is highest in the λC-equilibrium ($p_0^{\lambda C} = 4.25$), and lowest in the C-equilibrium ($p_0^C = 4.03$), with the price in the λ-equilibrium ($p_0^\lambda = 4.12$) being somewhere in the middle. In other words, whether we start in a world with only unsecured or only secured debt, as we head into a world where agents have access to both, this financial innovation pushes asset prices up. This holds true even in a case where the financial innovation is not directly linked to the secured debt market, as in the case when we compare the C-economy to the λC-economy.

The next question is to consider under what sufficient conditions this result holds more generally. Once again, we start from the set of assumptions that hold together the regime in the λC-equilibrium. Then, we observe that the equilibria of the λ- and λC-economies are sensitive to the choice of $\underline{\lambda}$, whereas the equilibrium of the C-economy is not. In particular, for small $\underline{\lambda}$, the increased competition from easy access to unsecured debt pushes housing prices up in the λ- and λC-economies, whereas this effect is much more muted under high $\underline{\lambda}$. This leads us to the consensus that our pricing result would only hold for intermediate values of the penalty parameter.

Proposition 4. *Consider the economies $E_\lambda, E_C, E_{\lambda C}$ satisfying Assumptions C1-C2 and P1-P2. Then, $\exists \lambda_1, \lambda_2 > 0$ s.t., $\forall \underline{\lambda} \in [\lambda_1, \lambda_2]$,*

$$p_0^C \leq p_0^\lambda \leq p_0^{\lambda C}.$$

Proof. See Appendix A.3. □

6 Conclusions and Future Work

In this paper, I build a perfectly competitive general equilibrium model that can endogenously explain the coexistence of secured and unsecured debt, where the secured debt market is modeled after the literature on endogenous leverage (Dubey et al., 1995; Geanakoplos, 1997; Geanakoplos and Zame, 1997, 2002) and the unsecured debt market is based on Dubey et al. (2005). I use this model to demonstrate how inequality affects the portfolio choice between debt types, and the effect of coexistence on asset prices. As far as I am aware, a model featuring and endogenous choice between both kinds of debt in GE with incomplete, perfectly competitive markets is a contribution in and of itself.

I begin the paper by setting up a binomial two-period general equilibrium model, with agents having access to two goods - a numeraire consumption good and a perfectly durable non-financial asset - and menus of two kinds of debt, namely, secured debt backed by the use of the non-financial asset as collateral, and unsecured debt backed by a threat of punishment. I then proceed to show that this λC-economy is internally consistent by showing the existence of equilibria in this model under standard assumptions.

Next, I define a coexistence equilibrium in this economy as an equilibrium that involves active trade in both secured and unsecured debt. Using this definition, I proceed to identify sufficient conditions under which all equilibria of this model must feature coexistence. This relies on the ideas that secured debt offers better returns, and hence, all agents would prefer to first take out as much secured debt as they can, while in the presence of sufficient endowment heterogeneity, at least one agent would also want to take on unsecured debt as well.

Then, I present a simple numerical example to illustrate the features of the model, and use this example to demonstrate other features of the equilibria I am interested in. In particular, I construct an example in which richer agents hold more secured debt, and less unsecured debt, a pattern that is reflected in real world data. I use this example to present further sufficient conditions (in addition to those sufficient to guarantee coexistence) under which we obtain a coexistence equilibrium of the λC-economy that displays this property.

Finally, I use the same numerical example to compare the price of the non-financial asset across various economies that differ only on the basis of what financial markets are open to agents for trade. To be specific, I compare the equilibrium in the complete λC-economy to equilibria in models that are identical except that agents only have access to either secured debt (C-economy) or unsecured debt (λ-economy), but not both. This comparison tells us that, in my constructed example, moving from either of the single-debt-type economies to the λC-economy pushes up the price of the non-financial asset. This can be interpreted as an effect of financial innovation, if an indirect one in the case of moving from the C-economy to the λC-economy.

My model brings together two strands of literature that both deal with how debt repayment is encouraged, and helps explain the coexistence of secured and unsecured debt. This task is non-trivial in and of itself, given that I am combining two already complex models. In addition, these questions have some significance in the real world: secured and unsecured debt coexist in significant proportions in consumer and corporate finance markets. Thus, this project may be a step towards an analysis of default in such generalized settings. The models also qualitatively explains the portfolio decisions of agents across the wealth distribution, whereby richer agents tend to hold more secured debt and less unsecured debt. Last but not least, the model also speaks to the potential spillover effects of financial innovation on asset prices.

To expand upon that last point, this paper fits into an agenda that explores how financial innovation affects pre-existing asset markets. In another working paper (Fostel et al., 2023), we explore the effect of financial innovation within the secured debt market on the price of assets used to back the secured debt. On the other hand, this paper demonstrates the effect of financial innovation introducing a new debt market (secured or unsecured) on the price of the asset used to back the secured debt contracts. This model, once developed further, can serve as a new starting point for this agenda, and may be helpful in answering other questions.

For example, the numerical example considered in this paper suggests that financial innovation of this kind (the introduction of unsecured debt markets to a world where previously only secured debt contracts were available for trade) has a clear redistributive effect on agents' welfare. In a secured-debt-only world, agents were limited in their ability to leverage the asset by their initial wealth; the ability to borrow using unsecured debt relaxes their budget constraint, and allows agents - especially the poor - to borrow, and therefore consume, more. While this increased demand also leads to higher prices, and therefore reduces utility across the board for borrowers, the positive effect of relaxing the budget constraint is strong enough for the poorer agents that the net result of the financial innovation is to make the poor better off at the expense of the rich.

On another note, the model would also serve to understand the effect of government policies that restrict or facilitate either kind of borrowing. For example, recent government policies that allow more gradual repayments of unsecured student loans may be interpreted as a reduction in the lower threshold of the penalty parameter, and thus, a comparative statics analysis using this model may shed some light on the spillover effects of such policy changes on goods and secured debt markets. For example, a quantitative extension of this model, once developed, could be useful in explaining the observed correlation between the rollout of these policies and the recent spike in housing prices.

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Appendices

A Proofs

A.1 Existence

Proof. **Setup:** We start by assuming that penalties are finite, $\lambda \in \mathbb{R}^I_+$. Fix a perturbation for the unsecured debt market, $(\epsilon_i)_{i\in I} >> 0$. This corresponds to a trembling-hand agent who buys and sells (for a net zero position) an amount ϵ_i of each unsecured debt i, and always fully delivers on his/her promises; this enables a refinement of the equilibrium concept to exclude cases where some unsecured debt pools go untraded simply because of undue pessimism regarding their repayment rates (Dubey et al., 1995). Fix a perturbation for the secured debt market, $\rho > 0$; this corresponds to bounding the promises made by all secured debt contracts from below, to exclude the case of some securities having zero deliveries. Fix a small lower bound, $b > 0$, to bound prices. To bound asset positions, fix an upper bound M s.t. more than M of any good is strictly better than twice of all endowments in the economy, i.e., $||(c, y)||_\infty > M \implies u^h(c, y) > u^h\left(2\sum_{h'\in H} e^{h'}_{c0}, 2\sum_{h'\in H} e^{h'}_{y0}\right), \forall h \in H$. Such an M exists w.l.o.g. under the assumptions made regarding utility functions in A1-4.

Price Simplex: Given the above setup, define the price simplex as

$$\Delta_b = \Bigg\{(p, (\pi_j)_{j\in J}, (\pi_i)_{i\in I}) \in \mathbb{R}^{S\times 2}_+ \times \mathbb{R}^J_+ \times \mathbb{R}^I_+ : p_{sc} + p_{sy} = 1, \forall s \in S; p_{sc}, p_{sy} \geq b, \forall s \in S;$$
$$\pi_i \in \left[0, \frac{1}{b}\right], \forall i \in I; \pi_j \in [0, 2], \forall j \in J\Bigg\}. \quad (1)$$

Note that the prices are normalized differently here than in the model described in the main body of the paper (where the normalization used is $p_{sc} = 1, \forall s \in S$), but since equilibria in this model are not independent of the chosen price level normalization, we can continue with the price simplex Δ_b to prove existence, and this existence result will continue to hold when the normalization described in the main text is chosen instead.

Truncated Choice Space: Next, I bound the space of positions of commodities, assets, and deliveries for agent h, $\square^h$, defined as

$$\square^h = \Big\{(c, y, \theta_j, \theta_i, \varphi_j, \varphi_i, D_j, D_i) \in \mathbb{R}^S_+ \times \mathbb{R}^S_+ \times \mathbb{R}^J_+ \times \mathbb{R}^I_+ \times \mathbb{R}^J_+ \times \mathbb{R}^I_+ \times \mathbb{R}^{S\times J}_+ \times \mathbb{R}^{S\times I}_+ :$$

$$||(c, y)||_\infty \leq M; \varphi^h_i \leq Q_i; \theta^h_i \leq 2 \sum_{h'\in H} Q_i;$$

$$||D||_\infty \leq ||Q||_\infty ||R||_\infty; \varphi^h_j \leq \sum_{h'\in H} e^h_y 0; \theta^h_j \leq \sum_{h'\in H} e^h_y 0\Big\}. \quad (2)$$

Then, the truncated choice space for the entire economy can be defined as the Cartesian product of the indiviudal truncated choice spaces of the agents,

$$\square^H \equiv \prod_{h\in H} \square^H.$$

Space of Potential Equilibria: Then, given $S^* \equiv \{U, D\}$, a potential equilibrium can be denoted as

$$\eta \equiv (p, (\pi_j)_{j\in J}, (\pi_i)_{i\in I}, K, (c^h, y^h, \theta^h_j, \theta^h_i, \varphi^h_j, \varphi^h_i, D^h_j, D^h_i)_{h\in H}) \in \Delta_b \times [0, 1]^{S^*\times I} \times \square^H \equiv \Omega_b. \quad (3)$$

Expected Delivery Map: Consider the mapping $\bar{K}_b : \Omega_b \to [0, 1]^{S^*\times I}$ that denotes the expected delivery rates of unsecured debt contracts in this economy with the trembling-hand agent,

$$\bar{K}_{bsi} = \begin{cases} \frac{p_s R_i \epsilon_i + \sum_{h\in H} p_s D^h_{si}}{p_s R_i \epsilon_i + \sum_{h\in H} p_s R_i \varphi^h_i}, & R_i \neq 0 \\ 1, & R_i = 0 \end{cases}. \quad (4)$$

This map is clearly continuous by construction of the trenbling hand.

Maximizing Value of Aggregate Excess Demand: Consider the correspondence of prices $\psi^0_b : \Omega_b \rightrightarrows \Delta_b$ that maximizes the value of aggregate excess demand,

$$\psi^0_b(\eta) = \arg \max_{(p,(\pi_j)_j,(\pi_i)_i)\in\Delta_b} \Big\{ p_{0c} \sum_{h\in H} \left(c^h_0 - e^h_0\right) + p_{0y} \sum_{h\in H} \left(y^h_0 - y^h\right) + \sum_j \pi_j \sum_{h\in H} \left(\theta^h_j - \varphi^h_j\right) + \pi_i \sum_{h\in H} \left(\theta^h_i - \varphi^h_i\right) +$$

$$\sum_{s\in S^*} \Big[p_{sc} \sum_{h\in H} \left(c^h_s - e^h_s\right) + p_{sy} \sum_{h\in H} \left(y^h_s - y^h_0\right) - \sum_i (1 - \bar{K}_{bsi}(\eta)) R_i \epsilon_i \Big] \Big\}. \quad (5)$$

Clearly, this correspondence is non-empty and convex-valued, and upper hemi-continuous (u.h.c.).

Optimal Choice Correspondence: Now, for each agent, define by $\psi^h_b : \Omega_b \rightrightarrows \square^h$ the correspondence that defines the optimal choice over the truncated budgeted set $B^h \bigcap \Box^h$,

$$\psi_b^0(\eta) = \arg \max_{(c,y,\theta_j,\theta_i,\varphi_j,\varphi_i,D_j,D_i)} [w^h(c, y, \theta_j, \theta_i, \varphi_j, \varphi_i, D_j, D_i, p) : (c, y, \theta_j, \theta_i, \varphi_j, \varphi_i, D_j, D_i) \in B^h \bigcap \Box^h]. \tag{6}$$

Note that this correspondence is non-empty-valued and convex-valued by the continuity and concavity of post-penalty utility. It is straightforward to show that the truncated budget set is continuous, in addition to being compact-valued since $B^h, \Box^h$ are both compact-valued. Further note that the post-penalty expected utility function is continuous by assumption. Given the above, we are looking at the argmax of a continuous function over a continuous, compact-valued correspondence, and Berge's Maximum Principle applies, implying that the correspondence ψ_b^h is u.h.c.

Equilibrium Correspondence: Define the equilibium correspondence $\psi_b : \Omega_b \Rightarrow \Omega_b$ as the product of these three correspondences,

$$\psi^b(\eta) = \psi_b^0(\eta) \times \{\bar{K}_b(\eta)\} \times \prod_{h \in H} \psi_b^h(\eta).$$

Kakutani's FPT: Since $\psi^b(\eta)$ is a u.h.c. correspondence with non-empty, convex values on a convex, compact subset of $\mathbb{R}^n$, by Kakutani's Fixed Point Theorem, this correspondence has a fixed point η^b.

Aggregate Excess Demand: Now, using a standard price player argument, we can show that there cannot be positive aggregate excess demand for any debt contract of either type. We can calculate the negative aggregate excess demand in some unsecured debt pools, and in the commodities, and show that they are functions of the arbitrary bound on prices, b, and go to 0 as $b \to 0$. For small enough b, the bounds are small enough that consumption is bounded by twice of everything in economy; but M of either commodity would be better, if it were feasible. This bounds commodity prices and unsecured debt prices as $b \to 0$.

Convergence: All variables of interest in the equilibrium object are bounded for small b, ρ. We can then take convergent subsequences as $b, \rho \to 0$ and find a limit point $\bar{E}$ which features (by taking limits on results above) non-positive aggregate excess demand, artificial bounds that do not bind, and with $\bar{E}$ being an optimum over the actual budget set. We can conclude that aggregate excess supply is not possible since the price player would be making negative profits. This gives us an equilibrium in the presence of the given trembling-hand agent. Further taking convergent subsequences as we let the influence of this agent disappear ($\epsilon \to 0$), we get a limit point that is a refined equilibrium for the economy. $\Box$

A.2 Coexistence

Proposition 5. *Consider a λC-economy that satisfies Assumptions 2-3. In such an economy, a coexistence equilibrium exists.*

Proof. We have already proved the existence of a refined equilibrium in a more general version of this economy. Furthermore, under assumptions C1 and C2.1, this equilibrium must involve trade in both the goods and financial markets; hence, if we prove that the above assumptions are sufficient conditions for the non-existence of refined equilibria where either only secured debt is being used or only unsecured debt is being used, we will have proved that a refined coexistence equilibrium exists, i.e. $\mathcal{E}(\omega) \neq \phi$.

We will begin by proving that any such equilibrium cannot be one where only secured debt is being used. To do this, we need only show that the secured debt contract that is being used cannot offer $100\%LTV$, which I do in Lemma 1. In the case of a non-financial asset, even though asset deliveries in future states of the world are pinned down exactly by their prices in those states such that the second term in the numerator on the LHS is zero, the first term $U_y^h(c_0^h, y_0^h)$ is necessarily non-zero since agents derive utility from the asset Y. Hence, the denominator, which is the difference between the price of the asset at time 0 and the amount that can be borrowed against it (i.e. the down payment), must be positive, and hence, we cannot reach $100\%LTV$. If this is the case, the poorest agent $h' \in H^B$ s.t. $e_0^{h'} = 0$ is unable to access secured debt, and will choose to use unsecured debt to finance their purchase of the house (or at least the down payment), since that is a better option than being excluded from the housing market. Note that this argument relies on Assumption C2.2.

Finally, we will prove that an equilibrium where everyone uses only unsecured debt cannot be sustained. Consider an economy in an unsecured-debt-only equilibrium. Consider the same agent $h' \in H$ s.t. $e_0^{h'} = 0$, who exists by Assumption C2.2. Let (c_0, y_0) denote the time-0 consumption allocations of this agent in this equilibrium, and let i_0 denote the amount agent h' borrows using unsecured debt in this equilibrium. Now, consider a unilateral deviation for this agent, from these allocations to a position where they also take on secured debt using the contract $j_0 = p_D$. Consider a deviation where the new consumption allocations are denoted by $\hat{c}_0, \hat{y}_0 = y_0$ such that

$$\frac{U_y^{h'}(\hat{c}_0, y_0) + \sum_{s \in S'} \mu_s^{h'} (p_s - \delta_s(p_D))}{p_0 - \pi_j} = \frac{U_c^{h'}(\hat{c}_0, y_0)}{1}. \tag{7}$$

Then, under the assumption that the bad state is not too bad (Assumption C2.3), we can show that $\hat{c}_0 < c_0$ (this boils down to requiring that $U_c^{h'}(c_0, y_0) > \sum_{s \in S'} \mu_s^{h'}$); the agent reduces their consumption of food, but maintains their consumption of housing at the same level as in the unsecured-only equilibrium. Let

$\hat{c}_0 = c_0 - \nu$; then, given that this agent is using secured debt to borrow $p_D y_0$, they can feasibly achieve this allocation by reducing their unsecured debt borrowing to $\hat{i}_0 = i_0 - p_D y_0 - \nu$.

Given the feasibility of this deviation, it remains to show that it is profitable. But this is directly seen by observing that reducing time-0 consumption implies a higher marginal utility of consumption at time 0.

$$\frac{U_y^{h'}(\hat{c}_0, y_0) + \sum_{s \in S'} \mu_s^{h'} (p_s - \delta_s(j))}{p_0 - \pi^j} = \frac{U_c^{h'}(\hat{c}_0, y_0)}{1} \tag{8}$$

$$> \frac{U_c^{h'}(c_0, y_0)}{1} \tag{9}$$

$$= \frac{U_y^{h'}(c_0, y_0) + \sum_{s \in S'} \mu_s^{h'} p_s}{p_0}, \tag{10}$$

where the inequality in (8) follows from the fact that $\hat{c}_0 < c_0$. This violates agent optimization in an unsecured-debt-only equilibrium; thus, we have built a feasible and profitable deviation for $h' \in H$, contradicting the existence of an unsecured-debt-only equilibrium.

Since an equilibrium exists in this economy, and the equilibrium can neither be secured-debt-only nor unsecured-debt-only, the equilibrium must feature coexistence. This proves that any equilibrium of this economy must be a coexistence equilibrium. □

A.3 Portfolio Choice

To Be Written

A.4 Asset Pricing

To Be Written